Drag Coefficients of Spheres in Continuum and Rarefied Flows

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Theme

ACCURATE representations of the drag coefficients of spheres over a wide range of flow conditions are a necessary prerequisite to the calculation of gas-particle flows. For greatest utility these representations should be in a form suitable for computer calculations. Two such correlations, both still used in computer programs, were published prior to the collection of a considerable body of experimental data, and, consequently are outdated. A third representation, published subsequently, requires the use of tabular data, and is inaccurate in some flow regimes of interest. This paper presents a new correlation, suitable for utilization in computer calculations, which simplifies in the limit to certain equations derived from theory, and which offers significantly improved agreement with the experimental data. The flow regimes of interest include continuum, slip, transition, and molecular flow at Mach numbers up to 6, and at Reynolds numbers up to the laminar-turbulent transition. The effect on drag of a temperature difference between the sphere and the gas is incorporated.

Content

The recommended set of correlating equations consists of one equation representing all of the subsonic flow regimes, a second equation representing the supersonic flow regimes at Mach numbers greater than 1.75, and a linear interpolation equation for the intervening region.

The subsonic equation is

$$C_D = 24\left[Re + S\left\{4.33 + \left(\frac{3.65 - 1.53\dfrac{T_w}{T}}{1 + 0.353\dfrac{T_w}{T}}\right) \times \exp\left(-0.247\frac{Re}{S}\right)\right\}\right]^{-1} + \exp\left(-\frac{0.5M}{\sqrt{Re}}\right)\left[\frac{4.5 + 0.38(0.03\,Re + 0.48\sqrt{Re})}{1 + 0.03\,Re + 0.48\sqrt{Re}} + 0.1M^2 + 0.2M^8\right] + \left[1 - \exp\left(-\frac{M}{Re}\right)\right]0.6\,S \tag{1}$$

where C_D is the drag coefficient, Re the Reynolds number (based on the sphere diameter, on Δv – the relative velocity between particle and sphere, – and on the freestream density and viscosity), M is the Mach number (based on Δv), S is the molecular speed ratio (equal to $M\sqrt{\gamma/2}$, where γ is the ratio of specific heats), T_w is the temperature of the sphere (assumed to be isothermal), and T is the temperature of the gas in the freestream.

Subsonic Continuum Flow: Equation (1) simplifies to the Stokes-Oseen equation ($C_D = 24\ Re^{-1} + 4.5$) at Reynolds numbers high enough to insure continuum flow, but low enough to avoid inertial effects. A new correlation which is valid up to the laminar-turbulent transition was derived to incorporate both inertial and compressibility effects (the portion within the brackets of the second term of Eq. (1). The inertial terms fit the so-called standard curve while the compressibility terms fit the data of Bailey and Hiatt. [1]

Subsonic Free Molecular Flow: The first term of Eq. (1) simplifies to the molecular flow equations of Langevin [2] and Epstein [3] at zero Reynolds number; values of 1.0 and 0.89 are used for the thermal accommodation coefficient, and the fraction of molecules reflected diffusely, respectively. The molecular flow limit of the third term, 0.6 S, upon addition to the Epstein and Langevin equations, approximates the compressible subsonic flow equations of Stalder and Zurick, [4]

Subsonic Slip and Transition Flow – Incompressible: A three constant equation of the form proposed by Millikin [5] is used in this region. One constant is selected to give the molecular flow equation in the limit. A second constant is selected to give agreement with the modified slip-flow equation of Millikan. [6] The third constant is used to obtain the best fit to Millikan's experimental data, which were obtained at Reynolds numbers spanning the entire slip and transition flow regimes.

Subsonic Slip and Transition Flow – Compressible: The exponential factors in the second and third form of Eq. (1) are derived to give agreement with the experimental data of Bailey and Hiatt.

The equation for the supersonic region at Mach numbers equal to or exceeding 1.75 is

$$C_D = \frac{0.9 + \dfrac{0.34}{M_\infty^2} + 1.86\left(\dfrac{M_\infty}{Re_\infty}\right)^{1/2}\left[2 + \dfrac{2}{S_\infty^2} + \dfrac{1.058}{S_\infty}\left(\dfrac{T_w}{T}\right)^{1/2} - \dfrac{1}{S_\infty^4}\right]}{1 + 1.86\left(\dfrac{M_\infty}{Re_\infty}\right)^{1/2}} \tag{2}$$

where the subscript ∞ refers to the freestream conditions.

Supersonic Molecular Flow: In the limit, as Reynolds number approaches zero, Eq. (2) simplifies to the term within brackets. This expression gives good agreement with the theoretical results of Stalder and Zurick.

Supersonic Continuum Flow: In the limit, as Mach number approaches zero, Eq. (2) simplifies to the first two terms in the numerator. This expression, derived as the sum of forebody drag and afterbody drag correlations, gives good agreement with the experimental data of Bailey and Hiatt.

Supersonic Slip and Transition Flow: In the intervening regions between continuum flow and molecular flow, Eq. (2) provides a correlation which is in good agreement with the experimental results of Bailey and Hiatt, which were obtained throughout the entire slip-flow regime and over a large portion of the transition regime.

In the supersonic region at Mach numbers between 1 and 1.75 the drag coefficients are linearly interpolated using the

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Table 1 Comparison of correlations with experimental drag coefficients

Data set	Range of M	Range of Re	Range of T_w/T_∞	No. of points	Maximum percentage deviation			
					Carlson	Crowe	Korkan	Henderson
Standard curve	–	$10^{-2}-10^{4}$	1	12	48	4	13	6
Molecular flow	$10^{-2}-6$	–	1	14	66	22	39	5
Slip and transition	$5\times10^{-7}-6\times10^{-6}$	$10^{-7}-10^{-5}$	1	50	9	13	23	3
Subsonic	$0.15-0.5$	$200-10^{4}$	1	19	53	21	15	8
Subsonic	$0.6-0.99$	$100-10^{4}$	1	31	54	28	18	15
Supersonic	$1.03-1.55$	$20-5\times10^{3}$	1	35	135	30	32	16
Supersonic	$1.75-2.55$	$5-5\times10^{3}$	1	14	117	31	14	13
Supersonic	$2.55-6$	$15-5\times10^{3}$	1	35	114	42	13	6
Supersonic	$3.5-6$	$10-100$	2-4	9	102	31	9	2

following equation

$$C_D(M_\infty,\ Re_\infty)=C_D(1.0,\ Re)+\frac{4}{3}(M_\infty-1.0)\left[C_D(1.75,\ Re_\infty)-C_D(1.0,\ Re)\right] \tag{3}$$

where C_D (1.0, Re) represents the coefficient calculated using Eq. (1) with $M=1.0$, and C_D (1.75, Re_∞) represents the coefficient calculated using Eq. (2) with $M_\infty=1.75$.

The accuracy of Eqs. (1-3) was tested and compared with correlations presented by Carlson and Hoglund,[7] by Crowe,[8] and by Korkan, Petrie and Bodonyi.[9] Nine sets of experimental and theoretical data were employed in this comparison. Three sources of experimental data were used: the standard curve data of Goldstein,[10] the extensive compilation of Bailey and Hiatt, and the very precise data of Millikan.[6] Theoretical molecular flow coefficients were calculated from the equations of Stalder and Zurick. For each data point, drag coefficients were calculated by each of the 4 correlations. For each data set, the maximum percentage deviation from the experimental or theoretical values was calculated. Results are given in Table 1.

The equation of Carlson and Hoglund is the simplest but also the least accurate with a maximum deviation of 117%. Crowe's correlating equations are both more complex and more accurate with a maximum deviation of 42%. Korkan's set of correlating equations is even more complex but gives an improved representation of the high Mach number data. The accuracy in the subsonic slip and transition regime is poorer, however, with a maximum deviation of 39%. Although more complex than the single equation of Carlson and Hoglund, the correlating equations presented in this paper are of equal complexity to that of Crowe's equations, and are somewhat simpler than Korkan's. The author's correlation is considerably more accurate than the other 3 with a maximum deviation of 16%. Drag coefficients at T_w/T_∞ ratios of 1, 2, 3, and 4 are represented with approximately equal accuracy.

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